Attention incoming AICE math students.

This packet contains the end of chapter question sets of the first three units of AICE math. The ***circled*** exercises represent knowledge that should be withing your grasp after completing Algebra 2 Honors.

They cover:

- Completing the square
- Non-linear simultaneous equations
- Vertex form of a parabola
- Quadratic inequalities
- Rational equations
- Inverse functions
- Composition of functions
- Equations of lines and linear systems
- Perpendicular bisectors
- Midpoint
- Distance formula
- Equations of circles

Necessary formulas are at the beginning of each section. Solutions are at the end of each section.

You should be fluent in the skills required for the circled exercises at the start of the school year. They are considered pre-requisite knowledge. There will be a test after the first 10 school days to assess your ability. The test questions will be taken directly from the circled question samples in this collection of exercises.

Please email Miska at thomas.miska@stjohns.k12.fl.us if you have any questions.

QUADRATICS CH 1

Quadratic equations can be solved by:

- factorisation
- completing the square
- using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Solving simultaneous equations where one is linear and one is quadratic

- Rearrange the linear equation to make either x or y the subject.
- Substitute this for x or y in the quadratic equation and then solve.

Maximum and minimum points and lines of symmetry

For a quadratic function $f(x) = ax^2 + bx + c$ that is written in the form $f(x) = a(x - h)^2 + k$:

- the line of symmetry is $x = h = -\frac{b}{2a}$
- if $a > 0$, there is a minimum point at (h, k)
- if $a < 0$, there is a maximum point at (h, k).

Quadratic equation $ax^2 + bx + c = 0$ and corresponding curve $y = ax^2 + bx + c$

- Discriminant $= b^2 - 4ac$.
- If $b^2 - 4ac > 0$, then the equation $ax^2 + bx + c = 0$ has two distinct real roots.
- If $b^2 - 4ac = 0$, then the equation $ax^2 + bx + c = 0$ has two equal real roots.
- If $b^2 - 4ac < 0$, then the equation $ax^2 + bx + c = 0$ has no real roots.
- The condition for a quadratic equation to have real roots is $b^2 - 4ac \geqslant 0$.

Intersection of a line and a general quadratic curve

- If a line and a general quadratic curve intersect at one point, then the line is a tangent to the curve at that point.
- Solving simultaneously the equations for the line and the curve gives an equation of the form $ax^2 + bx + c = 0$.
- $b^2 - 4ac$ gives information about the intersection of the line and the curve.

> 0	two distinct real roots	two distinct points of intersection
$= 0$	two equal real roots	one point of intersection (line is a tangent)
< 0	no real roots	no points of intersection

END-OF-CHAPTER REVIEW EXERCISE 1

1 A curve has equation $y = 2xy + 5$ and a line has equation $2x + 5y = 1$.

The curve and the line intersect at the points A and B. Find the coordinates of the midpoint of the line AB. [4]

2 a Express $9x^2 - 15x$ in the form $(3x - a)^2 - b$. [2]

b Find the set of values of x that satisfy the inequality $9x^2 - 15x < 6$. [2]

3 Find the real roots of the equation $\frac{36}{x^4} + 4 = \frac{25}{x^2}$. [4]

4 Find the set of values of k for which the line $y = kx - 3$ intersects the curve $y = x^2 - 9x$ at two distinct points. [4]

5 Find the set of values of the constant k for which the line $y = 2x + k$ meets the curve $y = 1 + 2kx - x^2$ at two distinct points. [5]

6 a Find the coordinates of the vertex of the parabola $y = 4x^2 - 12x + 7$. [4]

b Find the values of the constant k for which the line $y = kx + 3$ is a tangent to the curve $y = 4x^2 - 12x + 7$. [3]

7 A curve has equation $y = 5 - 2x + x^2$ and a line has equation $y = 2x + k$, where k is a constant.

a Show that the x-coordinates of the points of intersection of the curve and the line are given by the equation $x^2 - 4x + (5 - k) = 0$. [1]

b For one value of k, the line intersects the curve at two distinct points, A and B, where the coordinates of A are $(-2, 13)$. Find the coordinates of B. [3]

c For the case where the line is a tangent to the curve at a point C, find the value of k and the coordinates of C. [4]

8 A curve has equation $y = x^2 - 5x + 7$ and a line has equation $y = 2x - 3$.

a Show that the curve lies above the x-axis. [3]

b Find the coordinates of the points of intersection of the line and the curve. [3]

c Write down the set of values of x that satisfy the inequality $x^2 - 5x + 7 < 2x - 3$. [1]

9 A curve has equation $y = 10x - x^2$.

a Express $10x - x^2$ in the form $a - (x + b)^2$. [3]

b Write down the coordinates of the vertex of the curve. [2]

c Find the set of values of x for which $y \leqslant 9$. [3]

10 A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

i For the case where $k = 2$, the line and the curve intersect at points A and B.

Find the distance AB and the coordinates of the mid-point of AB. [5]

ii Find the two values of k for which the line is a tangent to the curve. [4]

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11 A curve has equation $y = x^2 - 4x + 4$ and a line has the equation $y = mx$, where m is a constant.

i For the case where $m = 1$, the curve and the line intersect at the points A and B.

Find the coordinates of the mid-point of AB. **[4]**

ii Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. **[5]**

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12 i Express $2x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. **[4]**

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at the points P and Q.

It is given that the coordinates of P are $(3, 7)$.

ii Find the coordinates of Q. **[3]**

iii Find the equation of the line joining Q to the mid-point of AP. **[3]**

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End-of-chapter review exercise 1

1 $\left(\frac{1}{2}, 0\right)$

2 a $\left(3x - \frac{5}{2}\right)^2 - \frac{25}{4}$ b $-\frac{1}{3} < x < 2$

3 $x = \pm 2, x = \pm\frac{3}{2}$

4 $x < -9 - 2\sqrt{3}$ or $x > -9 + 2\sqrt{3}$

5 $k < 1$ or $k > 2$

6 a $\left(1\frac{1}{2}, -2\right)$ b $k = -4$ or $k = -20$

7 a Proof b $(6, 29)$
c $k = 1, C = (2, 5)$

8 a Proof b $(2, 1), (5, 7),$
c $2 < x < 5$

9 a $25 - (x - 5)^2$ b $(5, 25)$
c $x \leqslant 1$ or $x \geqslant 9$

10 i $3\sqrt{5}, \left(-\frac{1}{2}, 5\right)$ ii $k = 3$ or 11

11 i $\left(2\frac{1}{2}, 2\frac{1}{2}\right)$ ii $m = -8, (-2, 16)$

12 i $2(x - 1)^2 - 1, (1, -1)$ ii $\left(-\frac{1}{2}, 3\frac{1}{2}\right)$
iii $y - 3 = -\frac{1}{5}(x - 2)$

FUNCTIONS CH2

Checklist of learning and understanding

Functions

- A function is a rule that maps each x value to just one y value for a defined set of input values.
- A function can be either one-one or many-one.
- The set of input values for a function is called the domain of the function.
- The set of output values for a function is called the range (or image set) of the function.

Composite functions

- $fg(x)$ means the function g acts on x first, then f acts on the result.
- fg only exists if the range of g is contained within the domain of f.
- In general, $fg(x) \neq gf(x)$.

Inverse functions

- The inverse of a function $f(x)$ is the function that undoes what $f(x)$ has done.
  $ff^{-1}(x) = f^{-1}f(x) = x$ or if $y = f(x)$ then $x = f^{-1}(y)$
- The inverse of the function $f(x)$ is written as $f^{-1}(x)$.
- The steps for finding the inverse function are:

  Step 1: Write the function as $y =$
  Step 2: Interchange the x and y variables.
  Step 3: Rearrange to make y the subject.
- The domain of $f^{-1}(x)$ is the range of $f(x)$.
- The range of $f^{-1}(x)$ is the domain of $f(x)$.
- An inverse function $f^{-1}(x)$ can exist if, and only if, the function $f(x)$ is one-one.
- The graphs of f and f^{-1} are reflections of each other in the line $y = x$.
- If $f(x) = f^{-1}(x)$, then the function f is called a self-inverse function.
- If f is self-inverse then $ff(x) = x$.
- The graph of a self-inverse function has $y = x$ as a line of symmetry.

Transformations of functions

- The graph of $y = f(x) + a$ is a translation of $y = f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$.
- The graph of $y = f(x + a)$ is a translation of $y = f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$.
- The graph of $y = -f(x)$ is a reflection of the graph $y = f(x)$ in the x-axis.
- The graph of $y = f(-x)$ is a reflection of the graph $y = f(x)$ in the y-axis.
- The graph of $y = af(x)$ is a stretch of $y = f(x)$, stretch factor a, parallel to the y-axis.
- The graph of $y = f(ax)$ is a stretch of $y = f(x)$, stretch factor $\frac{1}{a}$, parallel to the x-axis.

Combining transformations

- When two vertical transformations or two horizontal transformations are combined, the order in which they are applied may affect the outcome.
- When one horizontal and one vertical transformation are combined, the order in which they are applied does not affect the outcome.
- Vertical transformations follow the 'normal' order of operations, as used in arithmetic
- Horizontal transformations follow the **opposite** order to the 'normal' order of operations, as used in arithmetic.

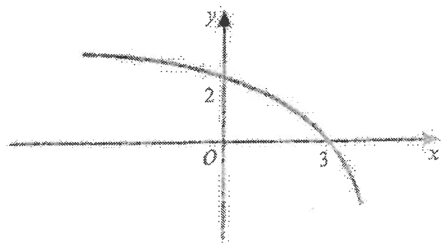
END-OF-CHAPTER REVIEW EXERCISE 2

1 Functions f and g are defined for $x \in \mathbb{R}$ by:

$$f : x \mapsto 3x - 1$$
$$g : x \mapsto 5x - x^2$$

Express $gf(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants. [5]

2



The diagram shows a sketch of the curve with equation $y = f(x)$.

a Sketch the graph of $y = -f\left(\frac{1}{2}x\right)$. [3]

b Describe fully a sequence of two transformations that maps the graph of $y = f(x)$ onto the graph of $y = f(3 - x)$. [2]

3 A curve has equation $y = x^2 + 6x + 8$.

a Sketch the curve, showing the coordinates of any axes crossing points. [2]

b The curve is translated by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, then stretched vertically with stretch factor 3.

Find the equation of the resulting curve, giving your answer in the form $y = ax^2 + bx$. [4]

4 The function $f : x \mapsto x^2 - 2$ is defined for the domain $x \geqslant 0$.

a Find $f^{-1}(x)$ and state the domain of f^{-1}. [3]

b On the same diagram, sketch the graphs of f and f^{-1}. [3]

5 i Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

The function $f : x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

ii State the smallest possible value of m for which f is one-one. [1]

iii For the case where $m = 5$, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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6 The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

i Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

ii State the range of f in terms of k. [1]

iii State the smallest value of p for which f is one-one. [1]

iv For the value of p found in part iii, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}, giving your answer in terms of k. [4]

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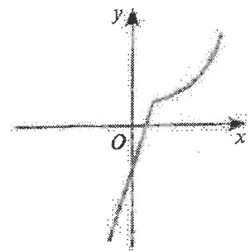
7



The diagram shows the function f defined for $-1 \leqslant x \leqslant 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leqslant x \leqslant 1, \\ \dfrac{4}{5-x} & \text{for } 1 < x \leqslant 4. \end{cases}$$

i State the range of f. [1]

ii Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

iii Obtain expressions to define the function f^{-1}, giving also the set of values for which each expression is valid. [6]

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8 The function f is defined by $f(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

i Express $f(x)$ in the form $a(x - b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = f(x)$. [4]

The function g is defined by $g(x) = 4x^2 - 24x + 11$, for $x \leqslant 1$.

ii State the range of g. [2]

iii Find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [4]

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9 i Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

ii The function f is defined by $f(x) = 2x^2 - 12x + 13$, for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. [1]

The value of k is now given to be 7.

iii Find the range of f. [1]

iv Find the expression for $f^{-1}(x)$ and state the domain of f^{-1}. [5]

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10 i Express $x^2 - 2x - 15$ in the form $(x + a)^2 + b$. [2]

The function f is defined for $p \leqslant x \leqslant q$, where p and q are positive constants, by

$$f : x \mapsto x^2 - 2x - 15.$$

The range of f is given by $c \leqslant f(x) \leqslant d$, where c and d are constants.

ii State the smallest possible value of c. [1]

For the case where $c = 9$ and $d = 65$,

iii find p and q, [4]

iv find an expression for $f^{-1}(x)$. [3]

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11 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

i Express $f(x)$ in the form $a(x - b)^2 - c$. [3]

ii State the range of f. [1]

iii Find the set of values of x for which $f(x) < 21$. [3]

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

iv Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

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12 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 1,$$

$$g : x \mapsto x^2 - 2.$$

i Find and simplify expressions for $fg(x)$ and $gf(x)$. [2]

ii Hence find the value of a for which $fg(a) = gf(a)$. [3]

iii Find the value of b $(b \neq a)$ for which $g(b) = b$. [2]

iv Find and simplify an expression for $f^{-1}g(x)$. [2]

The function h is defined by

$$h : x \mapsto x^2 - 2, \text{ for } x \leqslant 0.$$

v Find an expression for $h^{-1}(x)$. [2]

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13 Functions f and g are defined by

$$f : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$

$$g : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

i Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

ii State the range of f. [1]

iii State the domain of f^{-1}. [1]

iv Sketch on the same diagram the graphs of $y = f(x)$, $y = g(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [4]

v Find an expression for $f^{-1}(x)$. [3]

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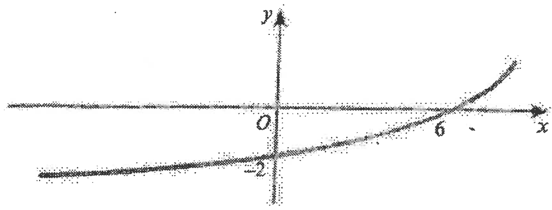
End-of-chapter review exercise 2

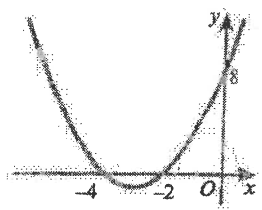
1 $\frac{25}{4} - 9\left(x - \frac{7}{6}\right)^2$

2 **a**



b Translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ followed by a reflection in the y-axis or reflection in the y-axis followed by translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

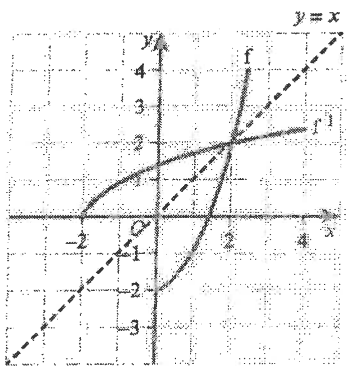
3 **a**



b $y = 3x^2 + 6x$

4 **a** $f^{-1}: x \mapsto \sqrt{x+2}$ for $x \geqslant -2$

b



5 **i** $-(x-3)^2 + 4$ **ii** 3

iii $f^{-1}(x) = 3 + \sqrt{4-x}$, domain is $x \leqslant 0$

6 **i** $(x-2)^2 - 4 + k$

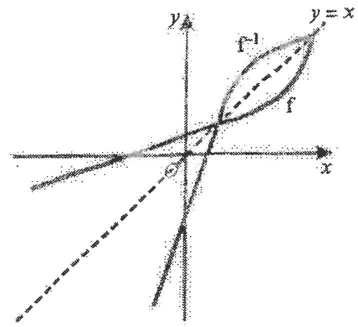
ii $f(x) \geqslant k - 4$

iii $p = 2$

iv $f^{-1}(x) = 2 + \sqrt{x + 4 - k}$, domain is $x \geqslant k - 4$

7 **i** $-5 \leqslant f(x) \leqslant 4$

ii



iii $f^{-1}(x) = \begin{cases} \frac{1}{3}(x+2) & \text{for } -5 \leqslant x \leqslant 1 \\ 5 - \frac{4}{x} & \text{for } 1 < x \leqslant 4 \end{cases}$

8 **i** $4(x-3)^2 - 25$, vertex is $(3, -25)$

ii $g(x) \geqslant -9$

iii $g^{-1}(x) = 3 - \frac{1}{2}\sqrt{x+25}$, domain is $x \geqslant -9$

9 **i** $2(x-3)^2 - 5$ **ii** 3

iii $f(x) \geqslant 27$

iv $f^{-1}(x) = 3 + \sqrt{\frac{x+5}{2}}$, domain is $x \geqslant 27$

10 **i** $(x-1)^2 - 16$ **ii** -16

iii $p = 6, q = 10$ **iv** $f^{-1}(x) = 1 + \sqrt{x+16}$

11 **i** $2(x-3)^2 - 11$ **ii** $f \geqslant -11$

iii $-1 < x < 7$ **iv** $k = 22$

12 **i** $fg(x) = 2x^2 - 3$, $gf(x) = 4x^2 + 4x - 1$

ii $a = -1$ **iii** $b = 2$

iv $\frac{1}{2}(x^2 - 3)$ **v** $h^{-1}(x) = -\sqrt{x+2}$

13 **i** $2(x-2)^2 + 2$ **ii** $2 \leqslant f(x) \leqslant 10$

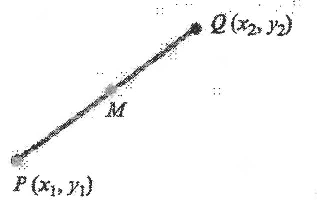
iii $2 \leqslant x \leqslant 10$

iv $f(x)$: half parabola from $(0, 10)$ to $(2, 2)$; $g(x)$: line through O at 45°; $f^{-1}(x)$: reflection of $f(x)$ in $g(x)$

v $f^{-1}(x) = 2 - \sqrt{\frac{1}{2}(x-2)}$

COORDINATE GEOMETRY Ch 13

Midpoint, gradient and length of line segment



- Midpoint, M, of PQ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- Gradient of PQ is $\frac{y_2 - y_1}{x_2 - x_1}$.
- Length of segment PQ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Parallel and perpendicular lines

- If the gradients of two parallel lines are m_1 and m_2, then $m_1 = m_2$.
- If the gradients of two perpendicular lines are m_1 and m_2, then $m_1 \times m_2 = -1$.

The equation of a straight line is:

- $y - y_1 = m(x - x_1)$, where m is the gradient and (x_1, y_1) is a point on the line.

The equation of a circle is:

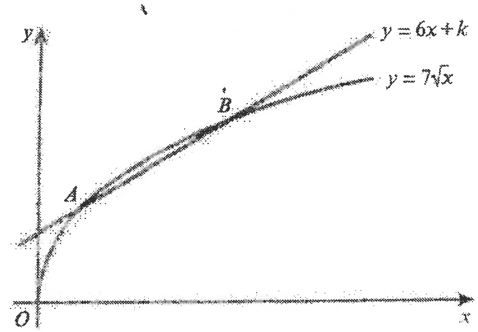
- $(x - a)^2 + (y - b)^2 = r^2$, where (a, b) is the centre and r is the radius.
- $x^2 + y^2 + 2gx + 2fy + c = 0$, where $(-g, -f)$ is the centre and $\sqrt{g^2 + f^2 - c}$ is the radius.

END-OF-CHAPTER REVIEW EXERCISE 3

1 A line has equation $2x + y = 20$ and a curve has equation $y = a + \frac{18}{x-3}$, where a is a constant.

Find the set of values of a for which the line does not intersect the curve. [4]

2



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant.

The curve and the line intersect at the points A and B.

i For the case where $k = 2$, find the x-coordinates of A and B. [4]

ii Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

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3 A is the point $(a, 3)$ and B is the point $(4, b)$.

The length of the line segment AB is $4\sqrt{5}$ units and the gradientis $-\frac{1}{2}$.

Find the possible values of a and b. [6]

4 The curve $y = 3\sqrt{x-2}$ and the line $3x - 4y + 3 = 0$ intersect at the points P and Q.

Find the length of PQ. [6]

5 The line $ax - 2y = 30$ passes through the points $A(10, 10)$ and $B(b, 10b)$, where a and b are constants.

a Find the values of a and b. [3]

b Find the coordinates of the midpoint of AB. [1]

c Find the equation of the perpendicular bisector of the line AB. [3]

6 The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

i Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

ii Show that the mid-point of PC lies on the line $y = x$. [4]

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7 The point P is the reflection of the point $(-7, 5)$ in the line $5x - 3y = 18$.

Find the coordinates of P. You must show all your working. [7]

8 The curve $y = x + 2 - \frac{4}{x}$ and the line $x - 2y + 6 = 0$ intersect at the points A and B.

a Find the coordinates of these two points. [4]

b Find the perpendicular bisector of the line AB. [4]

9 The line $y = mx + 1$ intersects the circle $x^2 + y^2 - 19x - 51 = 0$ at the point $P(5, 11)$.

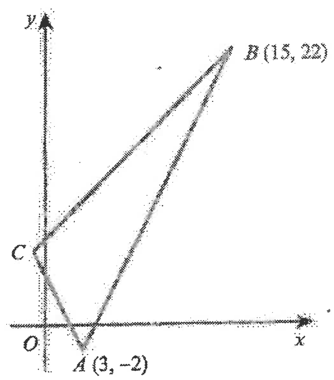
a Find the coordinates of the point Q where the line meets the curve again. [4]

b Find the equation of the perpendicular bisector of the line PQ. [3]

c Find the x-coordinates of the points where this perpendicular bisector intersects the circle.

Give your answers in exact form. [4]

10



The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB, AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

i Find the gradient of AB and deduce the value of m. [2]

ii Find the coordinates of C. [4]

The perpendicular bisector of AB meets BC at D.

iii Find the coordinates of D. [4]

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11 The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

i Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

ii A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. [5]

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12 The coordinates of A are $(-3, 2)$ and the coordinates of C are $(5, 6)$.

The mid-point of AC is M and the perpendicular bisector of AC cuts the x-axis at B.

i Find the equation of MB and the coordinates of B. [5]

ii Show that AB is perpendicular to BC. [2]

iii Given that $ABCD$ is a square, find the coordinates of D and the length of AD. [2]

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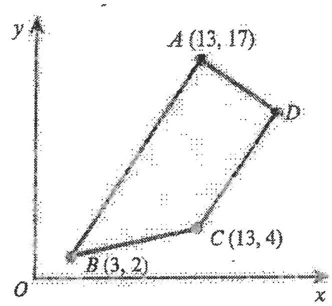
13 The points $A(1, -2)$ and $B(5, 4)$ lie on a circle with centre $C(6, p)$.

a Find the equation of the perpendicular bisector of the line segment AB. [4]

b Use your answer to part **a** to find the value of p. [1]

c Find the equation of the circle. [4]

14



$ABCD$ is a trapezium with AB parallel to DC and angle $BAD = 90°$.

a Calculate the coordinates of D. [7]

b Calculate the area of trapezium $ABCD$. [2]

15 The equation of a curve is $xy = 12$ and the equation of a line is $3x + y = k$, where k is a constant.

a In the case where $k = 20$, the line intersects the curve at the points A and B.

Find the midpoint of the line AB. [4]

b Find the set of values of k for which the line $3x + y = k$ intersects the curve at two distinct points. [4]

16 A is the point $(-3, 6)$ and B is the point $(9, -10)$.

a Find the equation of the line through A and B. [3]

b Show that the perpendicular bisector of the line AB is $3x - 4y = 17$. [3]

c A circle passes through A and B and has its centre on the line $x = 15$. Find the equation of this circle. [4]

17 The equation of a circle is $x^2 + y^2 - 8x + 4y + 4 = 0$.

a Find the radius of the circle and the coordinates of its centre. [4]

b Find the x-coordinates of the points where the circle crosses the x-axis, giving your answers in exact form. [4]

c Show that the point $A(6, 2\sqrt{3} - 2)$ lies on the circle. [2]

d Show that the equation of the tangent to the circle at A is $\sqrt{3}x + 3y = 12\sqrt{3} - 6$. [4]

End-of-chapter review exercise 3

1 $2 < a < 26$

2 i $\frac{4}{9}$ and $\frac{1}{4}$ ii $\frac{49}{24}$

3 $a = -4, b = -1$ or $a = 12, b = 7$

4 10

5 a $a = 5, b = -2$ b $(4, -5)$

c $y = -\frac{2}{5}x - 3\frac{2}{5}$

6 i $16t^2$ ii Proof

7 $(13, -7)$

8 a $(-2, 2), (4, 5)$ b $y = -2x + 5\frac{1}{2}$

9 a $(-2, -3)$ b $y = -\frac{1}{2}x + 4\frac{3}{4}$

c $\frac{19}{2} - \sqrt{113}, \frac{19}{2} + \sqrt{113}$

10 i $2, m = 1$ ii $(-1, 6)$

iii $(5, 12)$

11 i $y = 2x - 2$ ii $(0, -2), \left(\frac{8}{5}, \frac{6}{5}\right)$

12 i $y = -2x + 6, (3, 0)$ ii Proof

iii $(-1, 8), 2\sqrt{10}$

13 a $y = -\frac{2}{3}x + 3$ b $p = -1$

c $(x - 6)^2 + (y + 1)^2 = 26$

14 a $(19, 13)$ b 104

15 a $\left(\frac{10}{3}, 10\right)$ b $k < -12, k > 12$

16 a $y = -\frac{4}{3}x + 2$ b Proof

c $(x - 15)^2 + (y - 7)^2 = 325$

17 a $4, (4, -2)$ b $4 - 2\sqrt{3}, 4 + 2\sqrt{3}$

c Proof d Proof