Checklist of learning and understanding

Quadratic equations can be solved by:

- factorisation
- completing the square
- using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Solving simultaneous equations where one is linear and one is quadratic

- Rearrange the linear equation to make either x or y the subject.
- Substitute this for x or y in the quadratic equation and then solve.

Maximum and minimum points and lines of symmetry

For a quadratic function $f(x) = ax^2 + bx + c$ that is written in the form $f(x) = a(x - h)^2 + k$:

- the line of symmetry is $x = h = -\frac{b}{2a}$
- if $a > 0$, there is a minimum point at (h, k)
- if $a < 0$, there is a maximum point at (h, k).

Quadratic equation $ax^2 + bx + c = 0$ and corresponding curve $y = ax^2 + bx + c$

- Discriminant $= b^2 - 4ac$.
- If $b^2 - 4ac > 0$, then the equation $ax^2 + bx + c = 0$ has two distinct real roots.
- If $b^2 - 4ac = 0$, then the equation $ax^2 + bx + c = 0$ has two equal real roots.
- If $b^2 - 4ac < 0$, then the equation $ax^2 + bx + c = 0$ has no real roots.
- The condition for a quadratic equation to have real roots is $b^2 - 4ac \geqslant 0$.

Intersection of a line and a general quadratic curve

- If a line and a general quadratic curve intersect at one point, then the line is a tangent to the curve at that point.
- Solving simultaneously the equations for the line and the curve gives an equation of the form $ax^2 + bx + c = 0$.
- $b^2 - 4ac$ gives information about the intersection of the line and the curve.

$b^2 - 4ac$	Nature of roots	Line and parabola
> 0	two distinct real roots	two distinct points of intersection
$= 0$	two equal real roots	one point of intersection (line is a tangent)
< 0	no real roots	no points of intersection

END-OF-CHAPTER REVIEW EXERCISE 1

1 A curve has equation $y = 2xy + 5$ and a line has equation $2x + 5y = 1$.

The curve and the line intersect at the points A and B. Find the coordinates of the midpoint of the line AB. **[4]**

2 a Express $9x^2 - 15x$ in the form $(3x - a)^2 - b$. **[2]**

b Find the set of values of x that satisfy the inequality $9x^2 - 15x < 6$. **[2]**

3 Find the real roots of the equation $\frac{36}{x^4} + 4 = \frac{25}{x^2}$. **[4]**

4 Find the set of values of k for which the line $y = kx - 3$ intersects the curve $y = x^2 - 9x$ at two distinct points. **[4]**

5 Find the set of values of the constant k for which the line $y = 2x + k$ meets the curve $y = 1 + 2kx - x^2$ at two distinct points. **[5]**

6 a Find the coordinates of the vertex of the parabola $y = 4x^2 - 12x + 7$. **[4]**

b Find the values of the constant k for which the line $y = kx + 3$ is a tangent to the curve $y = 4x^2 - 12x + 7$. **[3]**

7 A curve has equation $y = 5 - 2x + x^2$ and a line has equation $y = 2x + k$, where k is a constant.

a Show that the x-coordinates of the points of intersection of the curve and the line are given by the equation $x^2 - 4x + (5 - k) = 0$. **[1]**

b For one value of k, the line intersects the curve at two distinct points, A and B, where the coordinates of A are $(-2, 13)$. Find the coordinates of B. **[3]**

c For the case where the line is a tangent to the curve at a point C, find the value of k and the coordinates of C. **[4]**

8 A curve has equation $y = x^2 - 5x + 7$ and a line has equation $y = 2x - 3$.

a Show that the curve lies above the x-axis. **[3]**

b Find the coordinates of the points of intersection of the line and the curve. **[3]**

c Write down the set of values of x that satisfy the inequality $x^2 - 5x + 7 < 2x - 3$. **[1]**

9 A curve has equation $y = 10x - x^2$.

a Express $10x - x^2$ in the form $a - (x + b)^2$. **[3]**

b Write down the coordinates of the vertex of the curve. **[2]**

c Find the set of values of x for which $y \leqslant 9$. **[3]**

10 A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

i For the case where $k = 2$, the line and the curve intersect at points A and B.

Find the distance AB and the coordinates of the mid-point of AB. **[5]**

ii Find the two values of k for which the line is a tangent to the curve. **[4]**

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11 A curve has equation $y = x^2 - 4x + 4$ and a line has the equation $y = mx$, where m is a constant.

i For the case where $m = 1$, the curve and the line intersect at the points A and B.

Find the coordinates of the mid-point of AB. **[4]**

ii Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. **[5]**

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12 **i** Express $2x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. **[4]**

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at the points P and Q.

It is given that the coordinates of P are $(3, 7)$.

ii Find the coordinates of Q. **[3]**

iii Find the equation of the line joining Q to the mid-point of AP. **[3]**

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Checklist of learning and understanding

Functions

- A function is a rule that maps each x value to just one y value for a defined set of input values.
- A function can be either one-one or many-one.
- The set of input values for a function is called the domain of the function.
- The set of output values for a function is called the range (or image set) of the function.

Composite functions

- $fg(x)$ means the function g acts on x first, then f acts on the result.
- fg only exists if the range of g is contained within the domain of f.
- In general, $fg(x) \neq gf(x)$.

Inverse functions

- The inverse of a function $f(x)$ is the function that undoes what $f(x)$ has done.
  $ff^{-1}(x) = f^{-1}f(x) = x$ or if $y = f(x)$ then $x = f^{-1}(y)$
- The inverse of the function $f(x)$ is written as $f^{-1}(x)$.
- The steps for finding the inverse function are:

  Step 1: Write the function as $y =$
  Step 2: Interchange the x and y variables.
  Step 3: Rearrange to make y the subject.

- The domain of $f^{-1}(x)$ is the range of $f(x)$.
- The range of $f^{-1}(x)$ is the domain of $f(x)$.
- An inverse function $f^{-1}(x)$ can exist if, and only if, the function $f(x)$ is one-one.
- The graphs of f and f^{-1} are reflections of each other in the line $y = x$.
- If $f(x) = f^{-1}(x)$, then the function f is called a self-inverse function.
- If f is self-inverse then $ff(x) = x$.
- The graph of a self-inverse function has $y = x$ as a line of symmetry.

Transformations of functions

- The graph of $y = f(x) + a$ is a translation of $y = f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$.
- The graph of $y = f(x + a)$ is a translation of $y = f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$.
- The graph of $y = -f(x)$ is a reflection of the graph $y = f(x)$ in the x-axis.
- The graph of $y = f(-x)$ is a reflection of the graph $y = f(x)$ in the y-axis.
- The graph of $y = af(x)$ is a stretch of $y = f(x)$, stretch factor a, parallel to the y-axis.
- The graph of $y = f(ax)$ is a stretch of $y = f(x)$, stretch factor $\frac{1}{a}$, parallel to the x-axis.

Combining transformations

- When two vertical transformations or two horizontal transformations are combined, the order in which they are applied may affect the outcome.
- When one horizontal and one vertical transformation are combined, the order in which they are applied does not affect the outcome.
- Vertical transformations follow the 'normal' order of operations, as used in arithmetic
- Horizontal transformations follow the **opposite** order to the 'normal' order of operations, as used in arithmetic.

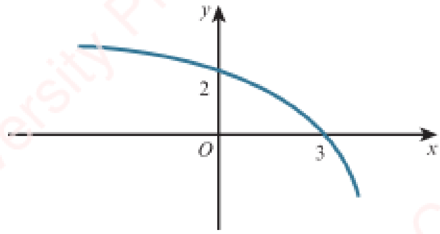
END-OF-CHAPTER REVIEW EXERCISE 2

1 Functions f and g are defined for $x \in \mathbb{R}$ by:

$$\mathrm{f} : x \mapsto 3x - 1$$
$$\mathrm{g} : x \mapsto 5x - x^2$$

Express $\mathrm{gf}(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants. **[5]**

2



The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

a Sketch the graph of $y = -\mathrm{f}\left(\frac{1}{2}x\right)$. **[3]**

b Describe fully a sequence of two transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(3 - x)$. **[2]**

3 A curve has equation $y = x^2 + 6x + 8$.

a Sketch the curve, showing the coordinates of any axes crossing points. **[2]**

b The curve is translated by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, then stretched vertically with stretch factor 3.

Find the equation of the resulting curve, giving your answer in the form $y = ax^2 + bx$. **[4]**

4 The function $\mathrm{f} : x \mapsto x^2 - 2$ is defined for the domain $x \geqslant 0$.

a Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

b On the same diagram, sketch the graphs of f and f^{-1}. **[3]**

5 **i** Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. **[3]**

The function $\mathrm{f} : x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

ii State the smallest possible value of m for which f is one-one. **[1]**

iii For the case where $m = 5$, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[4]**

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6 The function $\mathrm{f} : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

i Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. **[2]**

ii State the range of f in terms of k. **[1]**

iii State the smallest value of p for which f is one-one. **[1]**

iv For the value of p found in part **iii**, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}, giving your answer in terms of k. **[4]**

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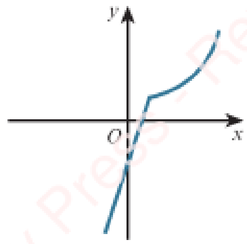
7



The diagram shows the function f defined for $-1 \leqslant x \leqslant 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leqslant x \leqslant 1, \\ \dfrac{4}{5 - x} & \text{for } 1 < x \leqslant 4. \end{cases}$$

i State the range of f. **[1]**

ii Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. **[2]**

iii Obtain expressions to define the function f^{-1}, giving also the set of values for which each expression is valid. **[6]**

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8 The function f is defined by $f(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

i Express $f(x)$ in the form $a(x - b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = f(x)$. **[4]**

The function g is defined by $g(x) = 4x^2 - 24x + 11$, for $x \leqslant 1$.

ii State the range of g. **[2]**

iii Find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. **[4]**

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9 i Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. **[3]**

ii The function f is defined by $f(x) = 2x^2 - 12x + 13$, for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. **[1]**

The value of k is now given to be 7.

iii Find the range of f. **[1]**

iv Find the expression for $f^{-1}(x)$ and state the domain of f^{-1}. **[5]**

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10 i Express $x^2 - 2x - 15$ in the form $(x + a)^2 + b$. **[2]**

The function f is defined for $p \leqslant x \leqslant q$, where p and q are positive constants, by

$$f : x \mapsto x^2 - 2x - 15.$$

The range of f is given by $c \leqslant f(x) \leqslant d$, where c and d are constants.

ii State the smallest possible value of c. **[1]**

For the case where $c = 9$ and $d = 65$,

iii find p and q, **[4]**

iv find an expression for $f^{-1}(x)$. **[3]**

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11 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

i Express $f(x)$ in the form $a(x - b)^2 - c$. **[3]**

ii State the range of f. **[1]**

iii Find the set of values of x for which $f(x) < 21$. **[3]**

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

iv Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. **[4]**

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12 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 1,$$

$$g : x \mapsto x^2 - 2.$$

i Find and simplify expressions for $fg(x)$ and $gf(x)$. **[2]**

ii Hence find the value of a for which $fg(a) = gf(a)$. **[3]**

iii Find the value of b $(b \neq a)$ for which $g(b) = b$. **[2]**

iv Find and simplify an expression for $f^{-1}g(x)$. **[2]**

The function h is defined by

$$h : x \mapsto x^2 - 2, \text{ for } x \leqslant 0.$$

v Find an expression for $h^{-1}(x)$. **[2]**

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13 Functions f and g are defined by

$$f : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$

$$g : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

i Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. **[3]**

ii State the range of f. **[1]**

iii State the domain of f^{-1}. **[1]**

iv Sketch on the same diagram the graphs of $y = f(x)$, $y = g(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. **[4]**

v Find an expression for $f^{-1}(x)$. **[3]**

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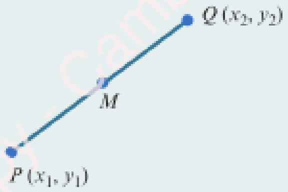
Checklist of learning and understanding

Midpoint, gradient and length of line segment



- Midpoint, M, of PQ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- Gradient of PQ is $\frac{y_2 - y_1}{x_2 - x_1}$.
- Length of segment PQ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Parallel and perpendicular lines

- If the gradients of two parallel lines are m_1 and m_2, then $m_1 = m_2$.
- If the gradients of two perpendicular lines are m_1 and m_2, then $m_1 \times m_2 = -1$.

The equation of a straight line is:

- $y - y_1 = m(x - x_1)$, where m is the gradient and (x_1, y_1) is a point on the line.

The equation of a circle is:

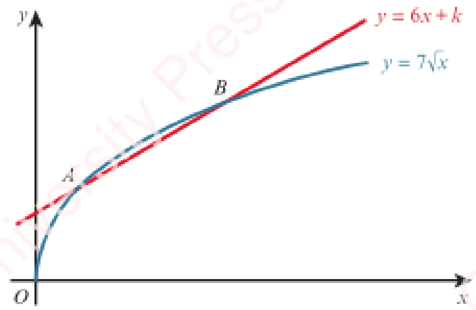
- $(x - a)^2 + (y - b)^2 = r^2$, where (a, b) is the centre and r is the radius.
- $x^2 + y^2 + 2gx + 2fy + c = 0$, where $(-g, -f)$ is the centre and $\sqrt{g^2 + f^2 - c}$ is the radius.

END-OF-CHAPTER REVIEW EXERCISE 3

1 A line has equation $2x + y = 20$ and a curve has equation $y = a + \frac{18}{x - 3}$, where a is a constant.

Find the set of values of a for which the line does not intersect the curve. **[4]**

2



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant.

The curve and the line intersect at the points A and B.

i For the case where $k = 2$, find the x-coordinates of A and B. **[4]**

ii Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. **[2]**

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3 A is the point $(a, 3)$ and B is the point $(4, b)$.

The length of the line segment AB is $4\sqrt{5}$ units and the gradientis $-\frac{1}{2}$.

Find the possible values of a and b. **[6]**

4 The curve $y = 3\sqrt{x - 2}$ and the line $3x - 4y + 3 = 0$ intersect at the points P and Q.

Find the length of PQ. **[6]**

5 The line $ax - 2y = 30$ passes through the points $A(10, 10)$ and $B(b, 10b)$, where a and b are constants.

a Find the values of a and b. **[3]**

b Find the coordinates of the midpoint of AB. **[1]**

c Find the equation of the perpendicular bisector of the line AB. **[3]**

6 The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

i Find the area of triangle AOB in terms of t. **[3]**

The line through P perpendicular to AB intersects the x-axis at C.

ii Show that the mid-point of PC lies on the line $y = x$. **[4]**

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7 The point P is the reflection of the point $(-7, 5)$ in the line $5x - 3y = 18$.

Find the coordinates of P. You must show all your working. **[7]**

8 The curve $y = x + 2 - \frac{4}{x}$ and the line $x - 2y + 6 = 0$ intersect at the points A and B.

a Find the coordinates of these two points. **[4]**

b Find the perpendicular bisector of the line AB. **[4]**

9 The line $y = mx + 1$ intersects the circle $x^2 + y^2 - 19x - 51 = 0$ at the point $P(5, 11)$.

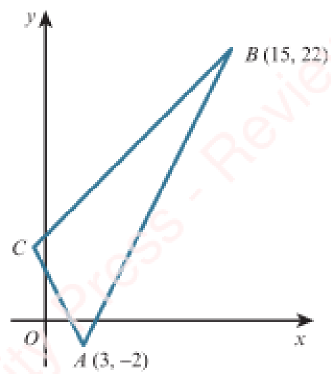
a Find the coordinates of the point Q where the line meets the curve again. **[4]**

b Find the equation of the perpendicular bisector of the line PQ. **[3]**

c Find the x-coordinates of the points where this perpendicular bisector intersects the circle.

Give your answers in exact form. **[4]**

10



The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB, AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

i Find the gradient of AB and deduce the value of m. **[2]**

ii Find the coordinates of C. **[4]**

The perpendicular bisector of AB meets BC at D.

iii Find the coordinates of D. **[4]**

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11 The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

i Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. **[4]**

ii A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. **[5]**

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12 The coordinates of A are $(-3, 2)$ and the coordinates of C are $(5, 6)$.

The mid-point of AC is M and the perpendicular bisector of AC cuts the x-axis at B.

i Find the equation of MB and the coordinates of B. **[5]**

ii Show that AB is perpendicular to BC. **[2]**

iii Given that $ABCD$ is a square, find the coordinates of D and the length of AD. **[2]**

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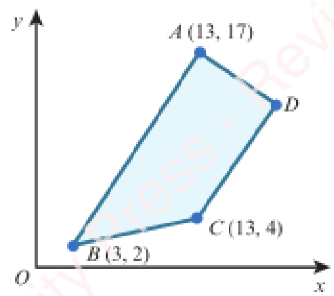
13 The points $A(1, -2)$ and $B(5, 4)$ lie on a circle with centre $C(6, p)$.

a Find the equation of the perpendicular bisector of the line segment AB. **[4]**

b Use your answer to part **a** to find the value of p. **[1]**

c Find the equation of the circle. **[4]**

14



$ABCD$ is a trapezium with AB parallel to DC and angle $BAD = 90°$.

a Calculate the coordinates of D. **[7]**

b Calculate the area of trapezium $ABCD$. **[2]**

15 The equation of a curve is $xy = 12$ and the equation of a line is $3x + y = k$, where k is a constant.

a In the case where $k = 20$, the line intersects the curve at the points A and B.

Find the midpoint of the line AB. **[4]**

b Find the set of values of k for which the line $3x + y = k$ intersects the curve at two distinct points. **[4]**

16 A is the point $(-3, 6)$ and B is the point $(9, -10)$.

a Find the equation of the line through A and B. **[3]**

b Show that the perpendicular bisector of the line AB is $3x - 4y = 17$. **[3]**

c A circle passes through A and B and has its centre on the line $x = 15$. Find the equation of this circle. **[4]**

17 The equation of a circle is $x^2 + y^2 - 8x + 4y + 4 = 0$.

a Find the radius of the circle and the coordinates of its centre. **[4]**

b Find the x-coordinates of the points where the circle crosses the x-axis, giving your answers in exact form. **[4]**

c Show that the point $A(6, 2\sqrt{3} - 2)$ lies on the circle. **[2]**

d Show that the equation of the tangent to the circle at A is $\sqrt{3}x + 3y = 12\sqrt{3} - 6$. **[4]**

Exercise 1G

1 **a** $0 \leqslant x \leqslant 3$ **b** $x < -2$ or $x > 3$

c $4 \leqslant x \leqslant 6$ **d** $-\frac{3}{2} < x < 2$

e $-6 \leqslant x \leqslant 5$ **f** $x < -\frac{1}{2}$ or $x > \frac{1}{3}$

2 **a** $x \leqslant -5$ or $x \geqslant 5$ **b** $-5 \leqslant x \leqslant -2$

c $x < -7$ or $x > 1$ **d** $-\frac{3}{2} \leqslant x \leqslant \frac{2}{7}$

e $\frac{4}{3} < x < \frac{5}{2}$ **f** $x < -4$ or $x > \frac{1}{2}$

3 **a** $-9 < x < 4$ **b** $x < 7$ or $x > 8$

c $-12 \leqslant x \leqslant 1$ **d** $-3 < x < 2$

e $x < -4$ or $x > 1$ **f** $-\frac{1}{2} < x < \frac{3}{5}$

g $x \leqslant -9$ or $x \geqslant 1$ **h** $x < -2$ or $x > 5$

i $-\frac{7}{2} < x < \frac{5}{3}$

4 $-3 < x < \frac{5}{2}$

5 **a** $5 \leqslant x < 7$ **b** $-7 \leqslant x < 1$

c $x < -2$ or $x \geqslant 3$

6 $x < -5$ or $x > 8$

7 **a** $1 < x \leqslant \frac{3}{2}$ **b** $-1 < x < 0$

c $-1 \leqslant x < 1$ or $x \geqslant 5$

d $-3 \leqslant x < 2$ or $x \geqslant 5$

e $-5 \leqslant x < -2$ or $1 \leqslant x < 2$

f $x < -4$ or $\frac{1}{2} \leqslant x < 5$

Exercise 1H

1 **a** Two equal roots **b** Two distinct roots

c Two distinct roots **d** Two equal roots

e No real roots **f** Two distinct roots

2 No real roots

3 $b = -2, c = -35$

4 **a** $k = \pm 4$ **b** $k = 4$ or $k = 1$

c $k = \frac{1}{4}$ **d** $k = 0$ or $k = 2$

e $k = 0$ or $k = -\frac{8}{9}$ **f** $k = -10$ or $k = 14$

5 **a** $k > -13$ **b** $k < \frac{57}{8}$

c $k < 2$ **d** $k < \frac{1}{2}$

e $k > \frac{3}{2}$ **f** $k < \frac{25}{16}$

6 **a** $k > \frac{1}{2}$ **b** $k > \frac{13}{12}$

c $k > \frac{26}{5}$ **d** $k > -\frac{39}{8}$

e $5 - \sqrt{21} < k < 5 + \sqrt{21}$

f $7 - 2\sqrt{10} < k < 7 + 2\sqrt{10}$

7 $k = \frac{p^2}{20}$

8 $k \leqslant \frac{25}{8}$

9 Proof

10 Proof

11 $k \leqslant -2\sqrt{2}$

Exercise 1I

1 $-5, -9$

2 $-1, 7$

3 5

4 **a** ± 10 **b** $(2, 4), (-2, -4)$

5 $-6, -2, (-1, 12), (1, 4)$

6 $k < -2$ or $k > 6$

7 $k < -4\sqrt{3}$ or $k > 4\sqrt{3}$

8 $k < 6$

9 $-3 < m < 1$

10 $k > 6$

11 $\frac{1}{2}$

12 Proof

13 Proof

End-of-chapter review exercise 1

1 $\left(\frac{1}{2}, 0\right)$

2 **a** $\left(3x - \frac{5}{2}\right)^2 - \frac{25}{4}$ **b** $-\frac{1}{3} < x < 2$

3 $x = \pm 2, x = \pm\frac{3}{2}$

4 $x < -9 - 2\sqrt{3}$ or $x > -9 + 2\sqrt{3}$

5 $k < 1$ or $k > 2$

6 a $\left(1\frac{1}{2}, -2\right)$ b $k = -4$ or $k = -20$

7 a Proof b (6, 29)
c $k = 1, C = (2, 5)$

8 a Proof b (2, 1), (5, 7),
c $2 < x < 5$

9 a $25 - (x - 5)^2$ b (5, 25)
c $x \leqslant 1$ or $x \geqslant 9$

10 i $3\sqrt{5}, \left(-\frac{1}{2}, 5\right)$ ii $k = 3$ or 11

11 i $\left(2\frac{1}{2}, 2\frac{1}{2}\right)$ ii $m = -8$, (−2, 16)

12 i $2(x - 1)^2 - 1$, (1, −1) ii $\left(-\frac{1}{2}, 3\frac{1}{2}\right)$
iii $y - 3 = -\frac{1}{5}(x - 2)$

2 Functions

Prerequisite knowledge

1 10

2 $3 - 2x$

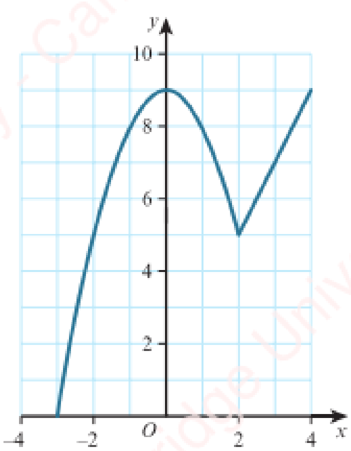
3 $f^{-1}(x) = \frac{x - 4}{5}$

4 $2(x - 3)^2 - 13$

Exercise 2A

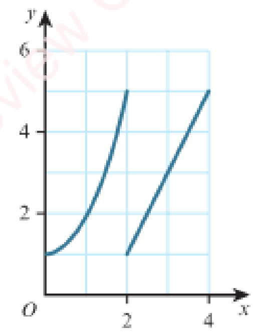
1 a function, one-one b function, many-one
c function, one-one d function, one-one
e function, one-one f function, one-one
g function, one-one h not a function

2 a



b Many-one

3 a



b each input does not have a unique output

4 a domain: $x \in \mathbb{R}, -1 \leqslant x \leqslant 5$
range: $f(x) \in \mathbb{R}, -8 \leqslant f(x) \leqslant 8$
b domain: $x \in \mathbb{R}, -3 \leqslant x \leqslant 2$
range: $f(x) \in \mathbb{R}, -7 \leqslant f(x) \leqslant 20$

5 a $f(x) > 12$ b $-13 \leqslant f(x) \leqslant -3$
c $-1 \leqslant f(x) \leqslant 9$ d $2 \leqslant f(x) \leqslant 32$
e $\frac{1}{32} \leqslant f(x) \leqslant 16$ f $\frac{3}{2} \leqslant f(x) \leqslant 12$

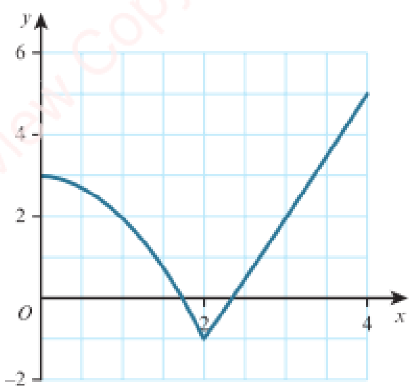
6 a $f(x) \geqslant -2$ b $3 \leqslant f(x) \leqslant 28$
c $f(x) \leqslant 3$ d $-5 \leqslant f(x) \leqslant 7$

7 a $f(x) \geqslant 5$ b $f(x) \geqslant -7$
c $-17 \leqslant f(x) \leqslant 8$ d $f(x) \geqslant 1$

8 a $f(x) \geqslant -20$ b $f(x) \geqslant -6\frac{1}{3}$

9 a $f(x) \leqslant 23$ b $f(x) \leqslant 5$

10 a



b $-1 \leqslant f(x) \leqslant 5$

11 $f(x) \geqslant k - 9$

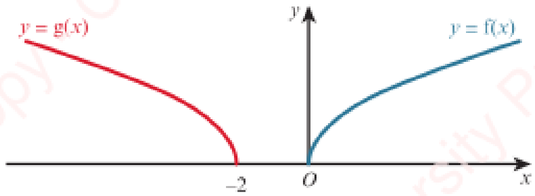
12 $g(x) \leqslant \frac{a^2}{8} + 5$

13 $a = 2$

14 $a = 1$ or $a = -5$

15 a $2(x - 2)^2 - 3$ b $k = 4$
c $x \in \mathbb{R}, -3 \leqslant x \leqslant 5$

11 Translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ followed by reflection in the y-axis or reflection in the y-axis followed by translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

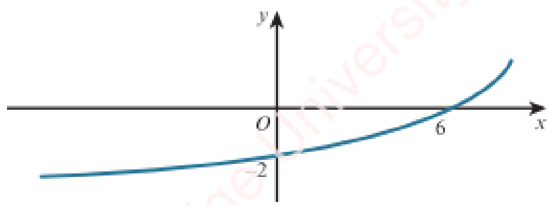


12 Translation $\begin{pmatrix} -10 \\ 0 \end{pmatrix}$ followed by a stretch parallel to the x-axis with stretch factor $\frac{1}{2}$ or stretch parallel to the x-axis with stretch factor $\frac{1}{2}$ followed by translation $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

End-of-chapter review exercise 2

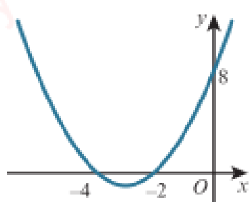
1 $\frac{25}{4} - 9\left(x - \frac{7}{6}\right)^2$

2 **a**



b Translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ followed by a reflection in the y-axis or reflection in the y-axis followed by translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

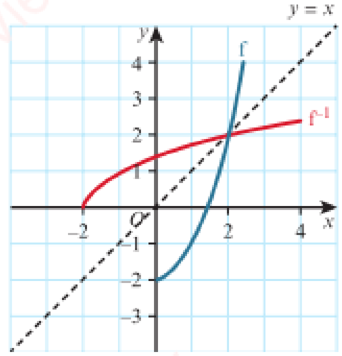
3 **a**



b $y = 3x^2 + 6x$

4 **a** $f^{-1} : x \mapsto \sqrt{x+2}$ for $x \geqslant -2$

b



5 **i** $-(x-3)^2 + 4$ **ii** 3

iii $f^{-1}(x) = 3 + \sqrt{4-x}$, domain is $x \leqslant 0$

6 **i** $(x-2)^2 - 4 + k$

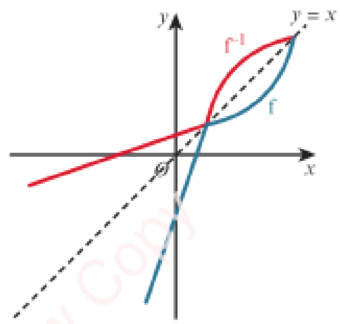
ii $f(x) \geqslant k - 4$

iii $p = 2$

iv $f^{-1}(x) = 2 + \sqrt{x + 4 - k}$, domain is $x \geqslant k - 4$

7 **i** $-5 \leqslant f(x) \leqslant 4$

ii



iii $f^{-1}(x) = \begin{cases} \frac{1}{3}(x+2) & \text{for } -5 \leqslant x \leqslant 1 \\ 5 - \frac{4}{x} & \text{for } 1 < x \leqslant 4 \end{cases}$

8 **i** $4(x-3)^2 - 25$, vertex is $(3, -25)$

ii $g(x) \geqslant -9$

iii $g^{-1}(x) = 3 - \frac{1}{2}\sqrt{x + 25}$, domain is $x \geqslant -9$

9 **i** $2(x-3)^2 - 5$ **ii** 3

iii $f(x) \geqslant 27$

iv $f^{-1}(x) = 3 + \sqrt{\frac{x+5}{2}}$, domain is $x \geqslant 27$

10 **i** $(x-1)^2 - 16$ **ii** -16

iii $p = 6, q = 10$ **iv** $f^{-1}(x) = 1 + \sqrt{x + 16}$

11 i $2(x-3)^2-11$ ii $f \geqslant -11$
iii $-1 < x < 7$ iv $k = 22$

12 i $fg(x) = 2x^2 - 3$, $gf(x) = 4x^2 + 4x - 1$
ii $a = -1$ iii $b = 2$
iv $\frac{1}{2}(x^2 - 3)$ v $h^{-1}(x) = -\sqrt{x+2}$

13 i $2(x-2)^2 + 2$ ii $2 \leqslant f(x) \leqslant 10$
iii $2 \leqslant x \leqslant 10$
iv f(x): half parabola from (0, 10) to (2, 2);
g(x): line through O at 45°;
$f^{-1}(x)$: reflection of f(x) in g(x)
v $f^{-1}(x) = 2 - \sqrt{\frac{1}{2}(x-2)}$

3 Coordinate geometry

Prerequisite knowledge

1 $\left(-4\frac{1}{2}, -2\right)$, 13

2 a $-\frac{1}{6}$ b 6

3 a $\frac{2}{3}$ b -5
c $7\frac{1}{2}$

4 a $(x-4)^2 - 21$ b $4 - \sqrt{21}$, $4 + \sqrt{21}$

Exercise 3A

1 a $PQ = 5\sqrt{5}$, $QR = 4\sqrt{5}$, $PR = 3\sqrt{5}$, right-angled triangle
b $PQ = \sqrt{197}$, $QR = \sqrt{146}$, $PR = 3\sqrt{5}$, not right angled

2 17 units^2

3 $a = 3$ or $a = -9$

4 $b = 3$ or $b = -5\frac{4}{5}$

5 $a = 2$, $b = -1$

6 a $(-2, -1)$ b $(-1, 9)$
c $2\sqrt{41}$, $2\sqrt{101}$

7 $k = 4$

8 $38\frac{1}{2}$ units^2

9 $k = 2$

10 $(-2, 6)$

11 a $(5, 2)$ b $8\sqrt{2}$

12 $A(-5, 5)$, $B(7, 3)$, $C(-3, -3)$

Exercise 3B

1 a $\frac{1}{5}, \frac{1}{6}$ b Not collinear

2 Proof

3 $-\frac{2}{5}, \frac{5}{2}$

4 $(7, -1)$

5 $k = \frac{5}{7}$

6 $k = 2$ or $k = 3$

7 $(0, -26)$

8 a 1 b 5

9 $a = 10$, $b = 4$

10 a $\frac{1}{2}$ b -2
c $a = 6$ or $a = -4$

11 a $(6, 6)$ b $a = -4$, $b = 16$, $c = 11$
c $4\sqrt{145}$ d 100

Exercise 3C

1 a $y = 2x + 1$ b $y = -3x - 1$
c $2x + 3y = 1$

2 a $2y = 3x - 3$ b $9x + 5y = 2$
c $2x - 3y = 9$

3 a $y = 3x + 4$ b $x + 2y = -8$
c $x + 2y = 8$ d $3x + 2y = 18$

4 a $y = 2x + 2$ b $5x + 3y = 9$
c $7x + 3y = -6$

5 $(8, 2)$

6 a $y = \frac{3}{2}x + 8$ b $(0, 8)$
c 39

7 a $(6, 3)$ b $y = -\frac{2}{3}x + 7$

8 a $y = \frac{4}{3}x + 10$ b $\left(-7\frac{1}{2}, 0\right)$, $(0, 10)$
c $12\frac{1}{2}$

9 a $2y = 5x + 33$ b 33

10 $E(4, 6)$, $F(10, 3)$

11 10

12 $(14, -2)$

13 a $y = -3x + 2$ b $(-1, 5)$
c $5\sqrt{10}, 4\sqrt{10}$ d 100

14 a i $y = 4\frac{1}{2}$ ii $x + y = 7$
b $\left(2\frac{1}{2}, 4\frac{1}{2}\right)$

15 a $y = 2x - 7$ b $\left(4\frac{2}{5}, 1\frac{4}{5}\right)$

16 $x + y = 8$, $3x + y = 3$. Other solutions possible.

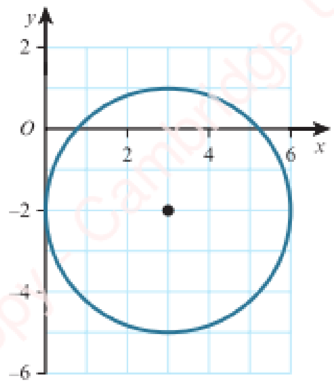
Exercise 3D

1 a $(0, 0), 4$ b $(0, 0), \frac{3\sqrt{2}}{2}$
c $(0, 2), 5$ d $(5, -3), 2$
e $(-7, 0), 3\sqrt{2}$ f $(3, -4), \frac{3\sqrt{10}}{2}$
g $(4, -10), \sqrt{6}$ h $\left(3\frac{1}{2}, 2\frac{1}{2}\right), 10$

2 a $x^2 + y^2 = 64$ b $(x - 5)^2 + (y + 2)^2 = 16$
c $(x + 1)^2 + (y - 3)^2 = 7$
d $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}$

3 $(x - 2)^2 + (y - 5)^2 = 25$

4 $(x + 2)^2 + (y - 2)^2 = 52$

5


6 $(x - 6)^2 + (y + 5)^2 = 25$

7 Proof

8 $(x - 5)^2 + y^2 = 8$ and $(x - 5)^2 + (y - 4)^2 = 8$

9 $(x - 4)^2 + (y - 2)^2 = 20$

10 $(x - 3)^2 + (y + 1)^2 = 16$, $(3, -1)$, 4

11 $y = \frac{3}{4}x - \frac{21}{2}$

12 $(x - 5)^2 + (y - 2)^2 = 29$

13 a Proof b $(x + 1)^2 + (y - 4)^2 = 20$

14 $(x - 5)^2 + (y + 3)^2 = 40$

15 $(x - 9)^2 + (y - 2)^2 = 85$

16 $(x + 3)^2 + (y + 10)^2 = 100$,
$(x - 13)^2 + (y + 10)^2 = 100$

17 a i $1 + \sqrt{2}$ ii Student's own answer
b i $3 + 2\sqrt{2}$ ii Student's own answer

Exercise 3E

1 $(-1, -4)$, $(5, 2)$

2 $2\sqrt{5}$

3 Proof

4 $-\frac{2}{29} < m < 2$

5 a $(0, 6), (8, 10)$ b $y = -2x + 16$
c $\left(5 - \sqrt{5}, 6 + 2\sqrt{5}\right), \left(5 + \sqrt{5}, 6 - 2\sqrt{5}\right)$
d $20\sqrt{5}$

6 $(4, 3)$

7 a $(x - 12)^2 + (y - 5)^2 = 25$ and
$(x - 2)^2 + (y - 10)^2 = 100$
b Proof

End-of-chapter review exercise 3

1 $2 < a < 26$

2 i $\frac{4}{9}$ and $\frac{1}{4}$ ii $\frac{49}{24}$

3 $a = -4, b = -1$ or $a = 12, b = 7$

4 10

5 a $a = 5, b = -2$ b $(4, -5)$
c $y = -\frac{2}{5}x - 3\frac{2}{5}$

6 i $16t^2$ ii Proof

7 $(13, -7)$

8 a $(-2, 2), (4, 5)$ b $y = -2x + 5\frac{1}{2}$

9 **a** $(-2, -3)$ **b** $y = -\frac{1}{2}x + 4\frac{3}{4}$

c $\frac{19}{2} - \sqrt{113}, \frac{19}{2} + \sqrt{113}$

10 **i** $2, m = 1$ **ii** $(-1, 6)$

iii $(5, 12)$

11 **i** $y = 2x - 2$ **ii** $(0, -2), \left(\frac{8}{5}, \frac{6}{5}\right)$

12 **i** $y = -2x + 6, (3, 0)$ **ii** Proof

iii $(-1, 8), 2\sqrt{10}$

13 **a** $y = -\frac{2}{3}x + 3$ **b** $p = -1$

c $(x-6)^2 + (y+1)^2 = 26$

14 **a** $(19, 13)$ **b** 104

15 **a** $\left(\frac{10}{3}, 10\right)$ **b** $k < -12, k > 12$

16 **a** $y = -\frac{4}{3}x + 2$ **b** Proof

c $(x-15)^2 + (y-7)^2 = 325$

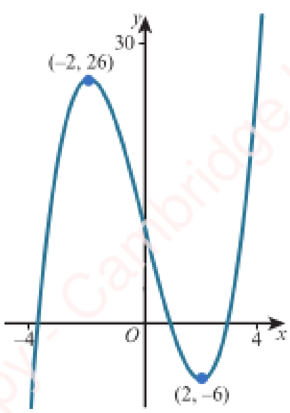
17 **a** $4, (4, -2)$ **b** $4 - 2\sqrt{3}, 4 + 2\sqrt{3}$

c Proof **d** Proof

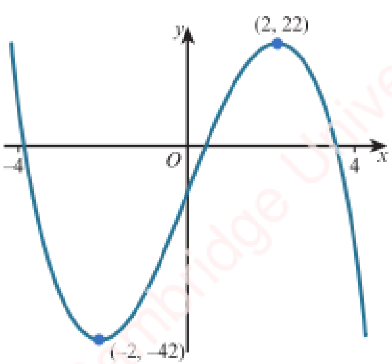
Cross-topic review exercise 1

1 $x = \pm\frac{2}{3}, x = \pm\frac{\sqrt{2}}{2}$

2 **a**



b



3 $a = 5, b = -2$

4 Translation $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$, vertical stretch with stretch factor 2

5 $y = -x^2 + 6x - 8$

6

7 **a** $1 \leqslant x \leqslant 5$ **b** $-13, 3$

8 $(k^2, -2k)$

9 $6\sqrt{5}$

10 **a** $k = 14$ **b** $y = \frac{1}{3}x + 4$

11 **a** $(-1, -11), (6, 3)$ **b** $k < -\frac{25}{12}$

12 **a** $k = -\frac{1}{2}$ **b** $x > 5$

13 **i** $\text{fg}(x) = 5x$, range is $\text{fg}(x) \geqslant 0$

ii $\text{g}^{-1}(x) = \frac{4 - 2x}{5x}$, domain is $0 < x \leqslant 2$

14 **a** $b = -5, c = -14$

b **i** $(2.5, -20.25)$ **ii** $-3 < x < 8$

15 **a** $2y = 3x + 25$ **b** $(-3, 8)$

16 **a** $36 - (x-6)^2$ **b** 36

c $x \leqslant 36, \text{g}^{-1}(x) \geqslant 6$ **d** $\text{g}^{-1}(x) = 6 + \sqrt{36 - x}$

17 **a** $3(x+2)^2 - 13$ **b** $(-2, -13)$

c $6 < x < 18$

18 **a** $a = 12, b = 2$ **b** -3

c $\text{g}^{-1}(x) = -3 + \sqrt{\frac{26 - x}{2}}$

19 **a** $(x-8)^2 + (y-3)^2 = 29$ **b** $5x + 2y = 75$